HEURISTIC METHODS FOR DESIGNING A GLOBAL POSITIONING SYSTEM SURVEYING NETWORK IN THE REPUBLIC OF SEYCHELLES

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الخلاصـــة

تزداد صعوبة إنشاء الشبكات المساحية المرصودة بالأقمار الصناعية الجي بي إس (GPS) مع ازدياد حجمها وبالتالي تصبح عملية تصميمها المبنية على الحل الفعَّـال بالغة الصعوبة . تحدَّد شبكة الجي بي إس المساحية برصد الاشارات أو القياسات الزمنية (session) المتشكلة بين نقاط التسوية (station) بواسطة أجهزة الاستقبال (receiver) المتموضعة على هذه النقاط . تبين هذه المقالة كيفية البحث عن التسلسل الأفضل لرصد هذه الإشارات بهدف الحصول على أفضل جَـدْوَلَة ممكنة (schedule) . تم استخدام الطرق التحسينية التقريبية (Heuristic methods) المبنية على برامج حاسوبية فعَّـالة لتصميم شبكات الجي بي إس المساحية الكبيرة ولتأمين الحلول المثالية أو القريبة من المثالية لهذه الشبكات . كما تم عرض العمليات الحسابية والنتائج الخالصة لإظهار فعالية وأداء الطرق التحسينية التقريبية المعتمدة وهما : طريقة التلدين التجريبي (simulated annealing) وطريقة البحث المحظور (tabu search) . لقد تم تطبيق هاتين الطريقتين على نفس الشبكة المساحية وذلك لإجراء المقارنة بين جودة الحلول الناتجة والسرعة الحسابية للحصول على هذه الحلول . استخدمت في هذه الدراسة (كمعلومات قياسية) خطة العمل الفعلية لرصد شبكة الجي بي إس المساحية المنفذَّة في جمهورية سيشل.

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ABSTRACT

The complexity of Global Positioning System (GPS) networks increases with their size and their design becomes highly difficult to solve effectively. A GPS network is a set of stations, co-ordinated by a series of sessions formed by placing receivers on the stations. This paper shows how to search for the best order in which to observe these sessions giving the best possible schedule. To solve this problem, heuristic techniques based on effective computer programs that provide an optimal or near optimal solution for large GPS networks are implemented. Computational experience and results are presented to show the effectiveness and performance of the proposed simulated annealing and tabu search heuristic techniques. The two heuristics are applied on the same network and compared with respect to solution quality and the execution times. The benchmark used was the actual operational schedule of a GPS network established in the Republic of Seychelles.

Keywords: Combinatorial Optimization Problem (COP), Global Positioning System (GPS), Heuristics, Simulated Annealing (SA), Tabu Search (TS).

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1. INTRODUCTION

The Global Positioning System (GPS) is an all-weather, global, satellite-based, continuously operating, positioning system developed and operated by the USA–Department of Defense. This system became available for the civilian surveying and navigation community in the early 1980s. The GPS operational constellation consists of 24 satellites orbiting the earth (see Figure 1) and provides the user with a 24-hour highly accurate three-dimensional position and timing system at most global locations [1]. The system is complicated but the method of using it in navigation is relatively simple. Satellites are orbiting the Earth as known reference points and the unknown positions of ground GPS receivers have to be determined. The satellites continuously transmit information of their positions and the receivers are able to measure their distance from the satellites at any time. If there are enough visible satellites the position of the receiver can be computed. The precision of this position depends on several factors such as the satellite orbit accuracy, the errors of the distance measurements, and artificial degradation of the signals by the military [2, 3]. The above processes which are involved in GPS surveying provide unprecedented accuracy at an economical rate compared to other surveying techniques. On the other hand, using the GPS technique is highly expensive and this becomes crucial as the amount of work increases for large networks.

Designing a GPS surveying network is a function of several operations such as planning, logistical considerations, reconnaissance, field procedures, and office calculation [4]. Within the field of Operational Research (OR), designing a GPS surveying network was previously based on a transformation of the GPS problem into a Traveling Salesman Problem (TSP) [5]. However, this approach remained limited to relatively small networks and was often unable to handle any additional constraints. More recently, heuristic techniques have been implemented that try to provide an optimal or near-optimal solution for designing large networks within an acceptable amount of computational effort [6]. These techniques have been developed to allow the formulation of a strategy for designing larger GPS networks which maximize the GPS technology benefit by reducing the total cost of carrying out the work.

This paper applies heuristic techniques to the GPS surveying problem and investigates their performance. The logistics of GPS surveying network problem and formulating it as a Combinatorial Optimization Problem (COP) will be outlined in Section 2. In Section 3 the proposed Simulated Annealing (SA) and Tabu Search (TS) techniques are discussed and their basic structures for a GPS surveying network are described. Section 4 reports the adopted criteria for selection of the control parameters and their effects on the performance of heuristic techniques. Also, the numerical results obtained

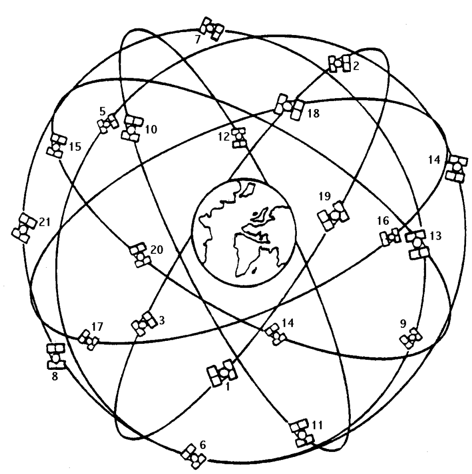


Figure 1. Global Positioning System satellite constellation [1].

by applying these techniques on a GPS surveying network established in the Seychelles are reported. Finally, the paper ends with conclusions and proposes avenues for future research.

2. SURVEYING NETWORK AND GPS LOGISTICS

The GPS surveying network, within the frame of heuristics, is a pure COP in which all variables are required to be integers. These kinds of combinatorial optimization are difficult to solve as their computational times increase exponentially with the number of variables. The network in GPS can be defined as a number of stations (*a*, *b*, *c*, *d*, *etc.*) which are co-ordinated by placing receivers (*X*, *Y*, *Z*, *etc.*) on them to determine sessions (*ab*, *ac*, *dc*, *etc.*) between them as shown in Figure 2. A session can be defined as a period of time during which two or more receivers simultaneously record satellite signals. The problem is to search for the best order in which these sessions can be organized to give the schedule of minimal cost. Thus, the schedule can be defined as a sequence of sessions to be observed consecutively. In practice this means determining how each GPS receiver (see Figure 3) should be moved between stations to be surveyed in an efficient manner taking into account important factors. For example, the efficiency of the GPS is a function of such factors as personnel availability, location of stations, sessions to be observed, receivers and satellites to be used [7], *etc.* Hence, the cost of a schedule is determined by routes of all receivers.

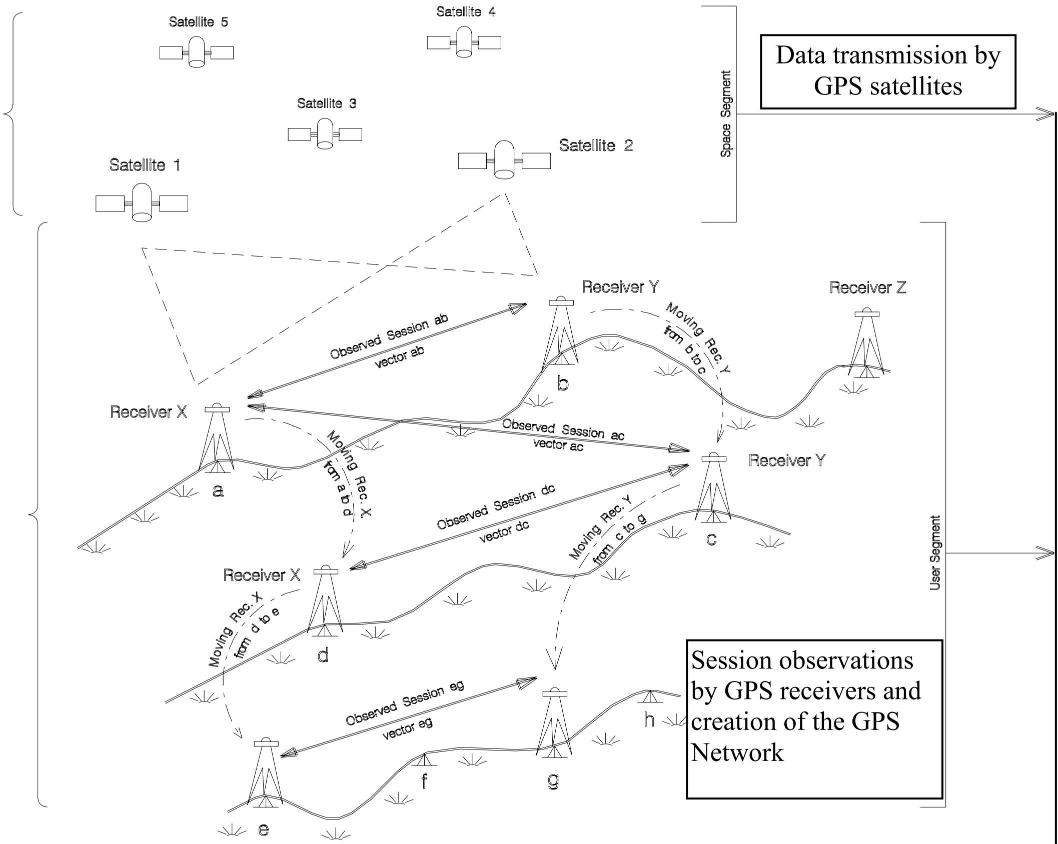
If n is the number of stations to be co-ordinated and r is the number of receivers to be used in the survey, then the minimum number of sessions to make the network solvable is given by $u_{\min} = \lceil (0.999 + n)/(r-1) \rceil$, where $\lceil \Psi \rceil$ is the smallest integer greater than Ψ. In this formula the minimum number of receivers should be at least two and 0.999 is added to build in redundancy into the network [8]. In practice, the actual number of sessions that is likely to be used to satisfy precision and reliability requirements would be $u_{\text{act}} = (2 \text{ to } 2.5)\, u_{\min}$. As can be seen, surveyors visit stations more than once (at least once) and observe more sessions than the minimum requirement. These additional observations, which are common throughout surveying, improve the precision and reliability of the resultant co-ordinates of the surveyed stations. With a large number of receivers, the time spent observing the network can be significantly reduced, because the minimum number of sessions falls as the number of receivers increases. For more details about the effect of the used number of receivers on required sessions to complete the network, the reader is highly recommended to consult Dare [8]. To represent the GPS surveying problem within the frame of combinatorial optimization, the following notation is used:

- C_{ij} : the cost of moving a receiver from station i to station j;
- N : the set of stations $N = \{1,\ldots,n\}$;
- R : the set of receivers $R = \{1,\ldots,r\}$;
- U : the set of sessions $U = \{1,\ldots,u\}$;
- S_p : the route of the receiver p in a schedule;
- $\sum C(S_p)$: the total cost of carrying out the survey of the whole network using all the receivers;
- $C(V)$: the total cost of a feasible schedule V.

The aim is to determine the optimal solution or close to it using heuristic techniques that satisfy the requirements of a GPS surveying network, *i.e.*,

$$\text{Minimize: } C(V) = \sum_{p \in R} C(S_p)\,.$$

An original cost matrix is constructed which represents the cost of moving a receiver from one station to another and computed based upon the criteria to be minimized. For example, as shown in Figure 2, the cost of observing session *ac* after observing session *ab* is obtained by the cost associated with moving receiver *Y* from station *b* to station *c* while receiver *X* remains at station *a*. Subsequently, the cost of observing session *dc* is obtained by the cost associated with moving receiver *X* from station *a* to station *d* while receiver *Y* remains at station *c*. The cost of observing session *eg* is obtained by the total cost associated with simultaneously moving receiver *X* from station *d* to station *e* and receiver *Y* from station *c* to station *g*. The cost could be evaluated purely upon time or purely upon distance. For more details about the evaluation of the cost matrix see Dare [8].



No.	**Session**	**Receiver X at:**		**Receiver Y at:**		**Receiver Z**			**Session**
q	u	Station n	Moving cost C_{ij}	Station n	Moving cost C_{ij}				**Cost**
1	*ab*	*a*	[- -]	*b*	[- -]				$w_1 = 0$
2	*ac*	*a*	[0]	*c*	$[C_{bc}]$				$w_2 = C_{bc}$
3	*dc*	*d*	$[C_{ad}]$	*c*	[0]				$w_3 = C_{ad}$
4	*eg*	*e*	$[C_{de}]$	*g*	$[C_{eg}]$				$w_4 = C_{de} + C_{eg}$
The total cost of the schedule									$\sum_{q=1} w_q$

Figure 2. Observation of sessions using GPS receivers.

GPS has the ability to perform precise surveying in a fraction of the time and cost than required by traditional methods. However, the time and cost to achieve this precision for networks can be optimized if the GPS logistics of the fieldwork is properly investigated. GPS logistics have been explained in Dare [8], but a brief summary of the logistics of GPS fieldwork is illustrated in Figure 2. The GPS network problem is practically considered as a complex COP as the number of possible sessions u increases as a function of n and r in the manner $u = n!/(n - r)!$ [9]. Hence, exact methods can only solve relatively small networks and are not practical as the size of the network increases [10, 11]. To provide a suitable method to design large networks more efficiently and effectively, the use of heuristic techniques has been developed.

3. THE GPS–HEURISTIC TECHNIQUES

Heuristic techniques, within the field of OR, are often based on ideas from Artificial Intelligence (AI). These techniques have been used successfully to find optimal or near optimal solutions to complex real-life problems in a reasonable amount of time [12–14]. The key objective of this paper is to find an effective solution for a GPS surveying network operable in the least time and at or near to the lowest cost using heuristic techniques. The fundamental concepts of any iterative heuristic approach consist of: the representation and construction of an initial solution; the generation of neighboring solutions; the acceptance strategy; and the stopping criteria. An iterative heuristic technique starts with an initial solution (within this context, an initial schedule). It iteratively attempts to improve upon the current schedule by a series of local improving changes (called moves) created by a suitably defined mechanism until a stopping criterion is met. A move can be defined as a transition from one schedule to another. Heuristic techniques differ in the way choices are made at each step during the process. SA derives from the physical sciences, while TS stems from the general tenets of problem solving. These are described briefly in the following sections and in more detail in Saleh [15].

Figure 3. The Wild GPS system 200 receiver.

3.1. The GPS–SA Technique

SA is a heuristic technique which imposes different randomized search, acceptance and stopping criteria on the local search method in order to escape poor quality minima inherent in this method. It has been successfully applied to a large number of problems related to Management Science (MS) as well as to engineering problems [16–19]. This technique permits occasional uphill moves with a probability controlled by a parameter called temperature T. Specifically, in each iteration, this technique creates a number of neighbors of the current schedule; if the neighbor has lower cost, then the technique moves to it, otherwise it moves to another neighbor with a probability $P(\Delta, T)$. The P depends on the cost differential Δ and on an adjustable temperature T. During the course of the cooling process, the temperature is adjusted starting from a high value (which yields higher probability of an uphill move) and tending towards zero as the number of iterations increases. The structure of the developed GPS–SA heuristic technique is shown in Figure 4.

3.2. The GPS–TS Technique

TS is based on procedures designed to cross boundaries of feasibility or local optimality, which were previously treated as barriers. This technique, which was introduced by Glover [20, 21], has been applied to several COPs whose applications range from graph theory to general pure and mixed integer programming problems [22–24]. A fundamental element underlying TS is the use of a tabu list. This list consists of a memory structure to control the search process by prohibiting moves that have recently been swapped. This technique moves from one schedule to another in order to reduce the cost of the schedules visited. When the technique visits a local optimum it does not terminate, but moves beyond it by choosing the best possible neighbor. The basic GPS–TS procedure is shown as in Figure 5.

4. HEURISTIC IMPLEMENTATION AND COMPUTATIONAL EXPERIENCE

Implementation of the developed heuristic techniques require specification of some particular components such as an initial schedule, neighborhood structure, and control parameters. The behavior of these techniques has been analyzed by execution of the program on a number of different types of GPS networks, varying in size, in Malta, and the Seychelles. For both networks, the developed heuristic techniques yielded good schedules. In this paper, SA and TS techniques applied to the GPS surveying network in the Seychelles are designed, developed, implemented, analyzed, and investigated by varying their control parameters, as shown in the following sections. The emphasis of the investigation is on how quickly each of the developed techniques found the best schedule. It is usual in applying heuristics to evaluate the proposed techniques by comparisons with known optimal solutions with respect to the solution quality and computational effort [25]. In this paper, however, an additional method is proposed.

4.1. Testing the Proposed Heuristic Performance

The performance of the GPS–SA and GPS–TS techniques is evaluated by monitoring the progress of the heuristic from the initial schedule until stopping criteria are met. A common performance measure to decide on the quality of a heuristic solution is the Relative Percentage Deviation (RPD) from the optimal solution. The RPD can be computed using Equation (1) as follows:

$$\mathrm{RPD} = \left\{ \frac{\text{Heuristic Solution} - \text{Optimal Solution}}{\text{Optimal Solution}} \right\} \times 100 \ . \tag{1}$$

Both the GPS–SA and GPS–TS techniques were tested on relatively small hypothetical and real GPS networks with known optimal schedules obtained by Dare [8]. The objective was to compare their performance with respect to the schedule quality. The data set for the hypothetical network consists of four stations ($n = 4$), two receivers ($r = 2$), and six sessions ($u = 6$). The real network consists of six stations ($n = 6$), two receivers ($r = 2$), and ten sessions ($u = 10$). The schedules obtained using the developed techniques had the same cost as the known optimal schedules [26] giving an RPD of zero. From the above, the ability of the developed heuristic techniques to quickly create high-quality schedules for relatively small GPS networks can be seen.

[I] <u>THE PROBLEM SPECIFIC DECISIONS</u>

(A) *FORMULATING* the original cost matrix:

Step 1 Insert the total number of stations, n.
Insert the estimated cost for each receiver's move.

(B) *CREATING* the actual cost matrix

Step 2 Insert the number of receivers, r.
Define the sessions to be observed, u.

(C) *DETERMINING* an initial schedule V_0:

Step 3 Create a feasible schedule V with cost $C(V)$ using the actual cost matrix.

[II] <u>THE PROBLEM GENERIC DECISIONS</u>

(D) *INITIALIZING* the cooling parameters:

Step 4 Set the initial starting value of the temperature, T.
Set the temperature length, L.
Set the cooling ratio, F ($F < 1$).
Set the number of iterations, K.

[III] <u>THE GENERATION MECHANISM</u>

(E) *SELECTING* and acceptance strategy of created neighbors I:

Step 5 Select a neighbor V' of V.
Let $C(V')$ = the cost of schedule V'.
Compute the move value $\Delta = C(V') - C(V)$.

Step 6 If $\Delta \le 0$ accept V' as a new schedule and set $V = V'$.
ELSE $\Delta > 0$.
IF $e^{-\Delta/T} > \theta$ set $V = V'$, where θ is a uniform random number $0 < \theta < 1$.
OTHERWISE retain the current schedule V.

(F) *UPDATING* the temperature:

Step 7 Update the annealing parameters using the cooling ratio
$T_{(k+1)} = F\ T_k$, where $\{k = 0, 1, 2\ldots\}$.

(G) *TERMINATING* the search:

Step 8 IF the stopping criterion is met THEN,
Show the output.
Declare the best obtained schedule.
Declare the computation time.
OTHERWISE.
Go to (**E**).

END.

Figure 4. The general outline for the GPS-SA procedure.

[I] INITIALIZATION:

(A) *FORMULATING* the original cost matrix:
Insert the total number of stations, n.
Insert the estimated cost for each receiver's move.

(B) *CREATING* the actual cost matrix:
Insert the number of receivers, r.
Define the sessions to be observed, u.

(C) *DETERMINING* an initial schedule V_0:
Create a feasible schedule V with cost $C(V)$.

(D) *INITIALIZING* the tabu parameters:
Set the Tabu List, $TL(u*u)$;
Set the Candidate Length, CL;
Set the Tabu Tenure, TT;
Set the iteration counter, $K=0$.

[II] SELECTION AND ACCEPTANCE STRATEGY

(E) *SELECTING* the best admissible move of cost $C(V_{\text{best}})$:
Create neighbors I of V_0;
Construct a CL for V_0 using I;
Improve V_0 into V_{best} with $C(V_{\text{best}})$.

(F) *UPDATING* the tabu parameters:
Build up the TL by adding $TL=[C(V_{\text{best}}),..]$;
V_{best} becomes the best possible new schedule;
Update the counter $K=K+1$, and the process continues.

[III] THE STOPPING CRITERIA:

(G) *TERMINATING* the search:
Stop if the stopping criterion is satisfied;
Given number of iterations, OR
Maximum number of iterations allowed without improving the best obtained schedule.
OTHERWISE.
Go to (**E**).

(H) *DECLARING* the output:
Declare the best obtained schedule;
Declare the computation time;
END.

Figure 5. The general outline for the GPS-TS procedure.

Measuring the performance of the proposed techniques on large networks for which the optimal schedules are not known may be quite difficult. An alternative method which has been adopted in this research which is to apply different heuristic techniques to a network with an available schedule of good quality [27]. The only good schedule is the actual operational schedule created manually using the skill and experience of the surveyors. This manually built schedule provides a benchmark that can enable solution comparisons to be made, and a means to evaluate the effectiveness of the proposed techniques is established. This enables us to see, very easily, how the schedule designed by the experienced surveyors can be improved upon. To generalize the above developed procedures and assist in the heuristic evaluation, tests were carried out using several large and different types of networks such as those established in Malta and the Seychelles. Although the general features of the developed techniques are similar, they have been tailored to meet the particular networks characteristics. This is because the networks have fundamentally different geometrical designs: triangular in Malta and linear in the Seychelles. However, apart from catering for this difference, the technique's principles are maintained. The comparison of the performance of these techniques on large and different networks was reported in Saleh [15].

4.2. The Seychelles GPS Network Design

This paper demonstrates the application of the developed techniques to the design of a GPS network in the Republic of Seychelles as shown in Figure 6. In August 1998, a team of surveyors from the School of Surveying at the University of East London in collaboration with the Land and Survey Division of the Ministry of Land Use and Habitat carried out a project to densify survey control points in the Seychelles. The principal requirement was to establish a densification network of survey points on Mahe, Praslin, and La Digue (the inner islands) using GPS receivers. This paper is based on Mahe only, as this was, by far, the largest of the three networks. The data set for this network consists of seventy-five stations (n = 75), three receivers (r = 3), and seventy-one sessions (u = 71). The stations used comprised of new UEL stations and already existing stations (MAPS stations). The fieldwork for this project was completed during a three-week period and the data processing for the observation was done in the School of Surveying at the University of East London.

The initial schedule V_0, with a cost of $C(V_0)$ = 994 minutes, was manually built using intuition and experience on a day-to-day basis whilst the survey work was carried out. At the end of one working day, a schedule for the following day was created. This schedule consisted of a sequence of 71 sessions, each of which used 3 receivers. The reason for using the manually built schedule as an initial starting schedule was that it was obtained by experience using the skill of the surveyors. In this way, an important practical evaluation of the developed techniques could be demonstrated. However, whatever criteria are adopted to select the initial schedule, the developed heuristic techniques are characteristically able to find better schedules. The manually built schedule and the GPS data, operational requirements, and objectives of this study including network logistics design and reconnaissance, can be seen in Dare [28].

One of the problems in the design of a method based on heuristic techniques is the trade-off between the size of neighborhood and the time used for exploration. The larger a neighborhood, the more comprehensive the search, but the longer the time spent for a single iteration. Thus, designing a good heuristic involves choosing a neighborhood that strikes the right balance [14]. In GPS surveying, a neighborhood of a schedule is normally defined as an alternative and discrete set of schedules which can be reached by a single move from the original schedule. In general, the neighborhood of a schedule $I(V)$ is a region of space surrounding this schedule and will contain some schedules which are worse and some which are better than the original one. In this research, the sequential neighborhood search structure for creating new schedules from the current schedule has been adopted and implemented. In this structure, the potential pair-swaps are examined in the order (1, 2), (1, 3), (1, 4),..., (1, u), (2, 3), (2, 4), ($u-1$, u), *etc.* The sequential structure is based on sessions-interchange. The change in cost is computed and the swap is accepted or rejected according to the acceptance strategy of the implemented techniques.

The behavior of these techniques on a number of trials on the Seychelles GPS network was investigated by varying the values of the control parameters. With the implementation of the GPS−SA technique, the overall cost of the initial schedule was reduced to 969 minutes as shown in Figure 7. Using the same initial schedule the overall cost was reduced to 933 minutes by implementing the GPS−TS technique as shown in Figure 8. The graphical depiction of the convergence to find the best possible schedule confirms the greediness property of heuristic techniques. GPS−TS is the

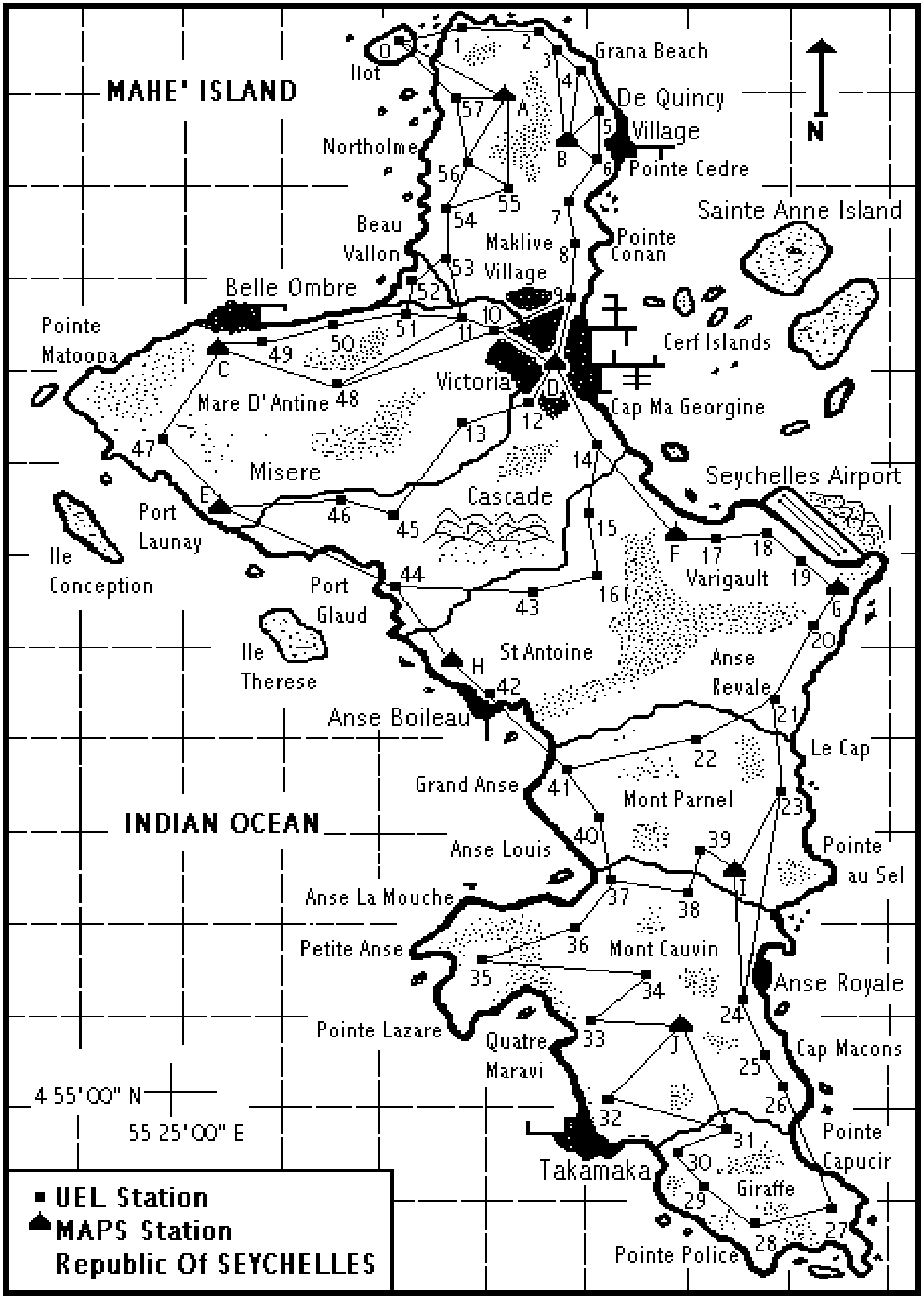


Figure 6. The Seychelles GPS surveying network.

most greedy technique, whereas the GPS–SA is the least greedy technique. The most useful measure for the evaluation of these techniques is the Relative Reduction of the Makespan (RRM) provided by these techniques with respect to the V_0, *i.e.*,

$$RRM = [(M_0 - M_{BFS}) / M_0]*100. \tag{2}$$

where

- RRM : The Relative Reduction of the Makespan.
- M_0 : The Makespan of the Initial Schedule V_0.
- M_{BFS} : The Makespan of the Best Found Schedule V_{BFS} by the GPS–heuristic techniques.
- K : Number of Iterations.
- ET : Execution Time in seconds.

The results of the comparison of the performance of both the GPS–SA and GPS–TS techniques are displayed in Table 1. The entries in the table are the relative reduction in the makespan computed from the schedules obtained by executing each technique, the number of iterations and the execution time to find the V_{BFS}. The GPS–SA technique run for 1700 seconds and completed 115 920 iterations to find the V_{BFS} with a cost of 969 minutes. In contrast, the running time for the GPS–TS technique to find the V_{BFS} with a cost of 933 minutes was 40 seconds for 20 iterations. In comparing the RRM, the GPS–TS technique finds better results and increases the RRM from 2.52% to 6.14%. It is noticeable from the above results that the RRM of the V_0 is close to the V_{BFS}. This is partly because of the limited number of reasonable choices of schedules for a linear-type network as in the Seychelles. Thus, a better schedule for a GPS surveying network can be found heuristically for triangular-type networks as in the Republic of Malta [29]. It is clear that this is because of the larger number of available states for each move unlike the linear-type network.

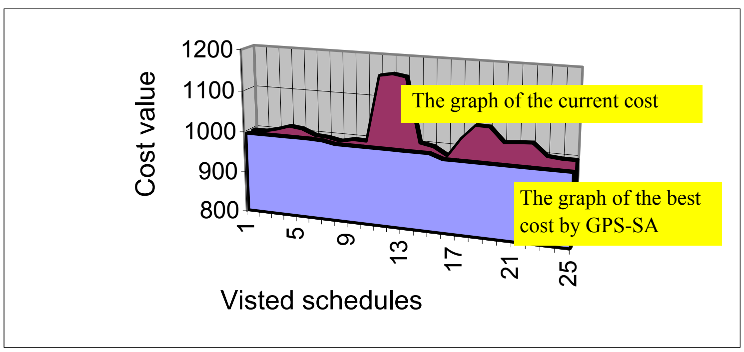


Figure 7. The current and V_{BFS} quality during cooling sequential search of GPS-SA applied to the GPS Seychelles network.

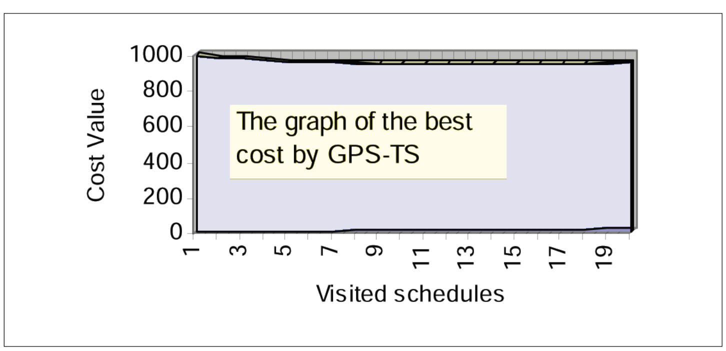


Figure 8. The graph of cost function values versus iteration number for the GPS–TS.

Table 1. Summary of the Computational Results for the Developed Techniques.

Technique	V_0	V_{BFS}	*RRM*%	*K*	*ET*
GPS-SA	994	969	2.52	115920	1700
GPS-TS	994	933	6.14	20	40

4.3. Investigation of the Control Parameters

This section, presents the computational results by applying the proposed heuristic techniques to the Seychelles GPS network using different annealing parameters. In order to implement these techniques, several control parameters have to be defined, tested, analyzed, and implemented. The *RRM* measure will be implemented in this section to evaluate the effect of the other control parameters on the obtained results in terms of the schedule quality and computational requirements. For heuristic techniques, to be accepted as a useful optimization model of the GPS network design, it is essential that the results obtained for tests such as the control parameters are in agreement with theory. This research illustrates agreement for these parameters and a summary of the results obtained can be seen in Table 2. In this section a brief discussion of the procedure for selecting heuristic parameters will be given. For further discussion and more detail about the adopted criteria for investigating and selecting these parameters, the reader is referred to Saleh [15].

Table 2. The Basic Control Parameters of the GPS–Heuristic Techniques.

GPS-SA technique	**GPS-TS technique**
Initial Temperature ***T*:198**	Tabu List ***TL*:71*71**
Temperature Updating Factor ***F*:0.85**	Tabu Tenure ***TT*:3**
Markov Chain Length ***L*:1242**	Candidate Length ***CL*:10**
Number of Iterations ***K*:115920**	Number of Iterations ***K*:20**

4.3.1. Investigation of the Annealing Parameters

The geometric cooling scale which suits the static nature of a GPS surveying network has been adopted using Equation (3) as follows:

$$T_{k+1} = F.\, T_k \,, \tag{3}$$

where

F : Temperature updating factor (a constant value smaller than but close to 1).

T_k : Current temperature.

T_{k+1} : New temperature.

The desire to increasingly inhibit uphill moves has led to using a geometric cooling scale. The scale is pair-wise constant, all jumps are downward, the ratio of successive costs of the session swap is a constant, and the temperature is bounded away from zero. In the GPS–SA implementation, as in any SA technique, it is very important to provide good and carefully chosen annealing parameters that must be defined according to the size of the designed network. These parameters (as shown in Figure 9) are: the initial value of temperature *T*, the temperature update factor *F*, the Markov chain length *L*, and the stopping criterion *K*.

4.3.1.1. Initial Temperature $\mathrm{T_i}$

The initial starting and final temperature for the Seychelles network was obtained by monitoring the evolution of the cost function during a number of rearrangements of a local search cycle which produces the largest and smallest possible change in the cost function values. The values of T_i = 1196 and T_f = 979 are set equal to the largest and smallest (non-zero) uphill steps found in 72 swaps. These swaps were randomly and arbitrarily performed before the GPS−SA started the optimization process. As shown in Figure 10, the values of the cost function are distributed over a number of distinct moves whose mutual distances are large compared to their cost and *vice versa* (bumpy topology). Determination of the start value of the temperature as described above can only be done reliably in small networks [15].

For large networks, an empirical method has been adopted to choose a value for T_i based on the ideas of Kirkpatrick *et al.* [30] and Johnson *et al.* [31]. The GPS–SA technique starts with an initial arbitrary value of T_i (say $T_i = 100$) using the intuition and experience of the user. This initial arbitrary value should preferably be close (1.5–2 times) to the number of observed sessions. The process of calculation of T_i can be done as follows. Ten moves are attempted at the given $T_i = 100$ in order to determine the acceptance ratio ($P_{\text{accep}} = 0.92$). P_{accep} is the fraction of the accepted moves ($m_{\text{accep}} = 138$ moves) to the total number of performed moves ($m_{\text{total}} = 150$ moves). To determine the change in the cost values Δ, the total cost of the performed moves ($C_{\text{total}} = 1305$ minutes) to the average for those moves which produced a positive increase in the cost function ($m_{\text{post}} = 79$ minutes) is calculated. More precisely, the equation of energy (4) has been solved to obtain a value for T_i as follows:

$$P = e^{-\Delta/T_i}\,. \tag{4}$$

Hence,

$$\ln(P) = \frac{-\Delta}{T_i} \Rightarrow T_i = \frac{-\Delta}{\ln(P)}\;.$$

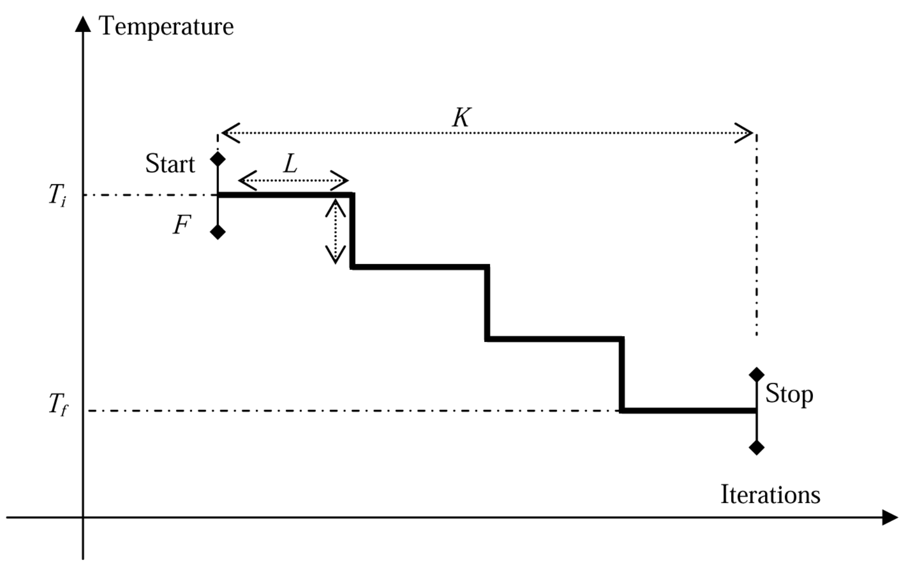


Figure 9. Parameters of the deterministic cooling scale.

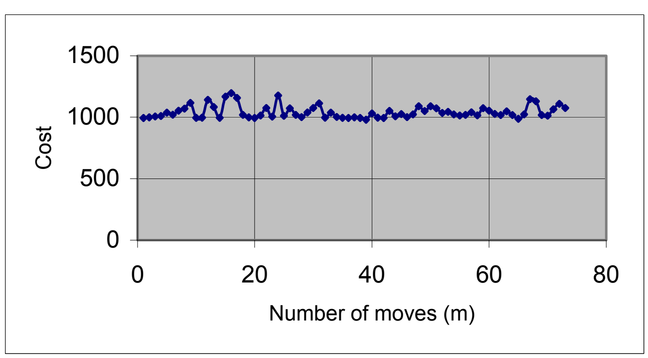


Figure 10. Topological evaluation of the cost function for the GPS Seychelles network.

Then,

$$P_{\text{accep}} = \frac{m_{\text{accep}}}{m_{\text{total}}} \Rightarrow P_{\text{accep}} = \frac{138}{150} = 0.92$$

$$\Delta = \frac{C_{\text{total}}}{m_{\text{post}}} \Rightarrow \Delta = \frac{1305}{79} = 16.5$$

$$T_i = \frac{-\Delta}{\ln(P)} \Rightarrow T_i = \frac{-16.5}{\ln(0.92)} = \frac{-16.5}{-0.08} \approx 198 \Rightarrow T_i = 198 .$$

Having computed T_i, the GPS–SA starts the cooling process with T_i = 198 reduced in a stepwise procedure using Equation (3). The best possible schedule is found for a fixed Markov chain length ($L = I(V) = 2415$) and temperature update factor ($F = 0.85$). The termination criterion is based on the changes of the schedule cost. If the cost of the last ten schedules are identical, then the annealing process is declared to be frozen; this happened when the stopping temperature equaled 12 (*i.e.*, the GPS−SA technique terminates at $T_{\text{stop}} = 12$).

Let us assume that a surveyor will need to design many similar networks on a regular basis. In this case, the surveyor wants to justify the overhead of initially performing trials over a range of temperatures to determine the optimum value of temperature T_{opt}. Within the literature of SA, a large number of theoretical search and suggested ideas have been carried out to define the T_{opt}, *e.g.*, Kirkpatrick *et al.* [30] and Connolly [32]. However, limited tests exploring these ideas failed to reliably estimate the empirical optimal values of the temperature.

In this paper, it was felt that a more refined scheme based on the maximization of the proportion of the search performed near T_{opt} is necessary to gain further improvements. In this scheme, the temperature is held fixed for a number of potential moves (sequentially chosen) before being dropped so that cooling takes place in a series of jumps. The cooling process continues until no change in the cost function is accepted at a particular temperature. The annealing parameters required by this scheme are T_i, the initial temperature, m, the number of moves examined at each temperature, and F, the cooling rate used to reduce the temperature.

The suggested optimization scheme (referred to here as SA.OPT) to determine the T_{opt} and improve the M_{BFS} can be stated as follows. By a suitable choice of the number of swaps examined in order, the surveyor can control the processing time of the GPS–SA technique. Thereafter, the temperature is controlled, then:

- the first down-hill move is selected;
- consecutive similar down-hill moves are rejected;

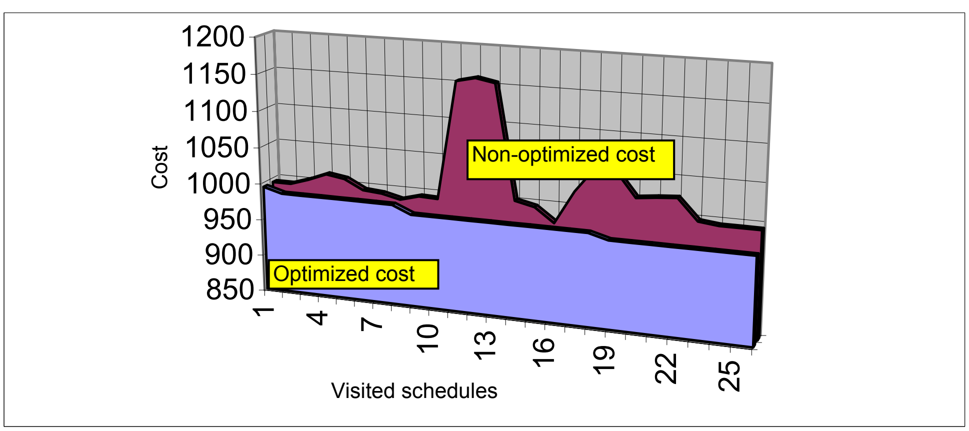


Figure 11. The current and V$_{BFS}$ *quality using GPS–SA.OPT scheme.*

- the next new down-hill move is accepted;
- the above steps are repeated until there is no improvement can be observed in the M_{BFS};
- the temperature T_{opt} is selected at which the current best schedule was found;
- the cooling process is stopped by setting $T_{\mathrm{best}} = T_{\mathrm{opt}}$.

The aim of this optimization scheme is to yield the most suitable temperature (T_{best}) that will be a reliable indicator of the T_{opt}. The optimization scheme has other practical features which make use of the flexibility inherent in SA to considerably reduce the overall computation efforts. This can be done by defining different sets of elementary swaps and different ranges of temperatures during the cooling process. Figure 11 shows that this scheme leads to an improved makespan with value of 969 minutes that deviates from the M_{BFS} by the GPS−SA technique by approximately 0.72%. The gain in performance with the suggested *cooling optimization scheme* is computed using Equation (5) as follows:

$$G = 100 \times \left\{ \frac{M_{\mathrm{SA}} - M_{\mathrm{SA.OPT}}}{M_{\mathrm{SA.OPT}}} \right\} = \frac{976 - 969}{969} \times 100 = 0.72\% \ , \tag{5}$$

where

M_{SA} : the makespan of the V_{BFS} by the GPS−SA;
$M_{\mathrm{SA.OPT}}$: the makespan of the V_{BFS} by the GPS−SA that optimized by SA.OPT;
G : the gain in the makespan by using the cooling optimization scheme.

Further improvement in the makespan can be achieved by adding post-analysis, which performs an efficient down-hill search (as shown in Figure 10) on the schedule found using this proposed scheme. With the temperature results presented in this section, it is possible to determine values of the other cooling parameters on the basis of these results. The next section presents results on the application of the GPS–SA technique in different situations. Other annealing parameters adopted in getting the V_{BFS} (as will be shown in the following section) are $F = 0.85$ and $L = 300$.

4.3.1.2. Markov Chain L *and Cooling Factor* F

The rule for decrementing the value of temperature F and the length of the Markov chain L, is related by the notion of equilibrium. In the literature, a large number of experiments on the relationship between the two parameters led to choosing specific values depending on the nature of the problem. Aarts and Van Laarhoven [33] developed a static cooling scale while Osborne and Gillett [34] implemented a dynamic cooling scale which suited their problems. In this research, L depends mainly on the size of the GPS surveying network. The adopted formula to obtain a suitable chain length is given in Equation (6) as follows:

$$L = \lambda * m \tag{6}$$

where m equals to the number of moves examined at each temperature and λ is a given number $\lambda = 1, 2, \ldots$ *etc.*, that can be chosen arbitrarily. However, using L and F leads to a time-complexity of the SA technique which is usually polynomial in the size of the network *i.e.*, proportional to the number of sessions in the initial schedule. So, as will be shown from the obtained results for the varying values of both L and F, the GPS−SA technique can be regarded as a polynomial-time approximation technique.

Figures 12 and 13 illustrate the behavior of the GPS–SA technique on a number of trials on the Seychelles network, F, varying in value from 0.95 to 0.70 and varying length, L, according to Equation (6). Each of the aforementioned figures shows a different L with a fixed F required to reach the best possible schedules. In Figure 12A, for example, the GPS–SA technique starts off with a given $L = M_{\mathrm{A}}$, where $M_{\mathrm{A}} = 10$, and carries out the cooling process until a schedule is found that has not changed over 10 subsequent chains (*i.e.*, local minimum). In Figures 12B, 12C, and 12D, the entire cooling process is repeated until the stopping criterion is met with different starting chain lengths $L_{\mathrm{B}} = 20$, $L_{\mathrm{C}} = 40$, and $L_{\mathrm{D}} = 80$ respectively and fixed $F = 0.95$. In Figures 12 and 13 the notations used are: BFC is the Best Found Cost by the SA technique for K iterations; L is the length of the Markov chain; and F is the cooling factor.

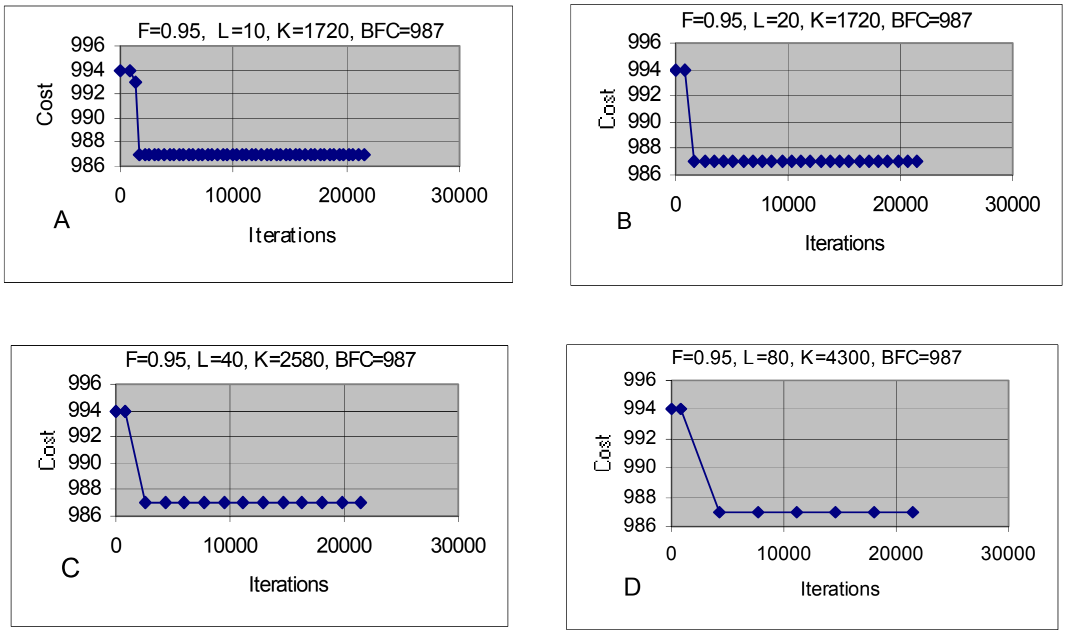


Figure 12. Evolution of the cost of observing the Seychelles GPS network while applying GPS–SA with varying L *and fixed* F = *0.95.*

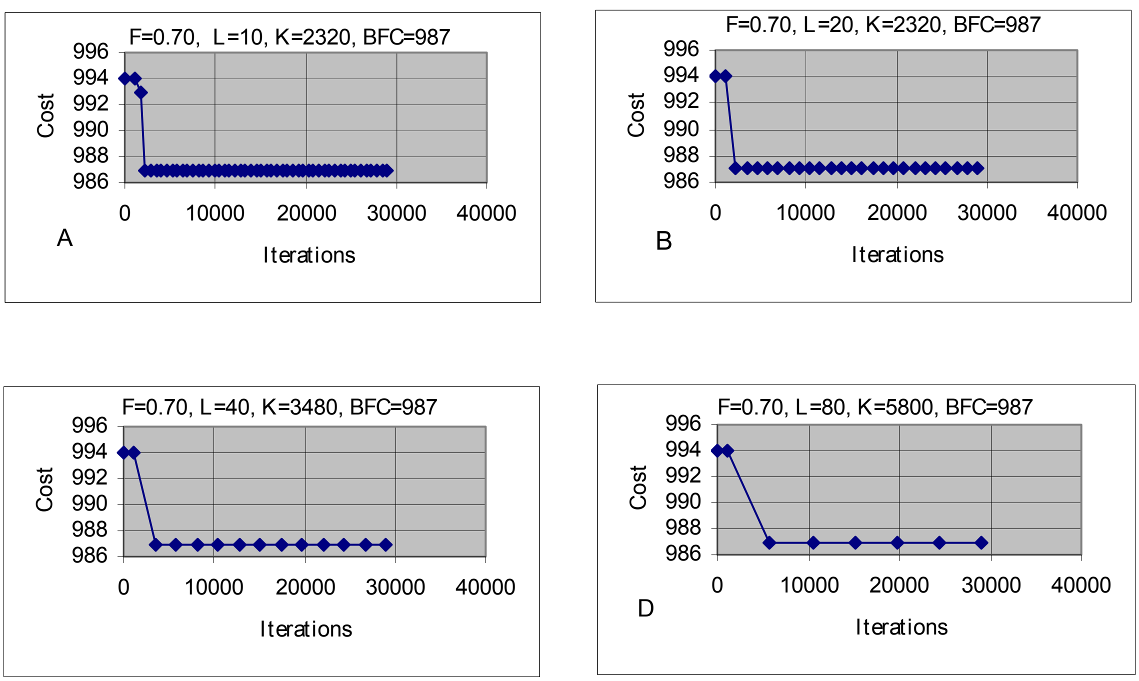


Figure 13. Evolution of the cost of observing the Seychelles GPS network while applying GPS–SA with varying L *and fixed* F = *0.70.*

By extending the above study to many examples, the experimental results for a fixed F, (say $F = 0.95$, and $L = 80$) reveal that the decrement of the temperature is not very critical provided the Markov chains are long enough. This length should allow the cost of the initial schedule to reach at best one schedule that is close to stationary cost. By reducing the length of the Markov chains, say to $L = 10$, the decrease of F becomes more critical. Using a similar fixed length of

Markov chain, say $L = 20$, and putting $F = 0.70$ makes the cooling process much slower than $F = 0.95$, but in both cases the same V_{BFS} is found. The obtained experimental values for this factor which control the behavior of the GPS–SA technique lie in the range from 0.80 to 0.90. These values are in agreement with the literature [30, 35]. The most practical annealing parameters for the Seychelles network are when $L_{max} = I$, (*i.e.*, it equals the size of the largest neighborhood $L = 1242$ when $F = 0.85$).

4.3.1.3. The Stopping Criterion H

The total processing time of the GPS–SA technique is calculated until the best possible schedules are found. The computational effort for the Seychelles GPS network with respect to the number of iterations (function of time) has been computed as follows. Each iteration of the cooling process requires four steps.

1. The search process has to be perturbed by creating (random or in order) a new schedule.
2. The difference in cost function must be calculated.
3. A decision must be made whether to adopt a new schedule.
4. The search process is updated if the new schedule is accepted.

The most time-consuming part of each iteration is the calculation of the cost function. Fortunately, for a GPS network, which is a static and not a dynamic issue, the calculation is identical for each iteration. The time-complexity of this technique is proportional to the number of sessions in the schedule. In other words, the total execution time increases as the networks become larger and the convergence to an optimal schedule or close to it becomes too slow. This result is in agreement with the literature [36, 37]. The stopping criterion is satisfied when the cost function's best value has not changed for the last tenth of the consecutive identical schedules ($H = 10$), as shown in Figure 7. The value of the V_{BFS} is not affected at all by increasing values of H from 10 to 100. From this it may be concluded that the stopping criterion based on the pre-specified number of H is reliable in those cases where the length of the Markov chain is very large. This has an obvious advantage of relating the stopping criterion to schedule changes at the cost of greater computational effort. From the above results, cycling in the GPS–SA technique is almost avoided due to randomness in the neighborhood search. On other hand, the GPS–SA technique is too slow in converging to an optimal solution or close to optimal.

4.4.2. Investigation of the Tabu Parameters

This section presents the computational results obtained by applying the GPS–TS technique to the Seychelles network using different tabu parameters. Tabu parameters consist of the Tabu List (*TL*), the Candidate List (*CL*), the Tabu Tenure (*TT*) and the stopping criterion (*H*). In order to avoid cycling, the move that turns back to the less expensive schedule just visited is prohibited. This forbidden task is accomplished by keeping the prohibited move tabu in a matrix structure [$u*u$] called the tabu list. The fixed size of *TL* is suitable for the statistical nature of the GPS network. For the Seychelles network, the selected tabu parameters (as shown in Table 2) for the V_{BFS} are $CL = 10$, $TT = 3$, $K = 20$, and $TL = 71*71$. The experimental results are summarized in the form of the bar charts as shown in the following sections.

4.4.2.1 Candidate List CL

The adopted *CL*, which suits the GPS surveying requirements, is a static "fixed length" that is specified by the user at the start of the running of the program. The length of the *CL* is dependent on the size of the network and experience. *CL* contains the potential moves that give the best possible neighbors surrounding the current schedule. By statically fixing the *CL*, the efficiency and speed of the search for the V_{BFS} is improved. For this application the behavior of the GPS–TS technique is investigated for fixed values of both *TT* and *K*, while increasing the value of the *CL*. The quality of the V_{BFS} does not seem to be affected at all by increasing the value of the *CL* parameters from 3 to 12. The main reason for this behavior seems to be the efficiency of the *CL* strategy. This strategy, for every tabu configuration search, discards most of the admissible moves and keeps only those leading to good schedules.

4.4.2.2. Tabu Tenure TT

The *TT* is another crucial parameter specified by the user, as discussed later, because it determines how restrictive the neighborhood search is. This parameter is also called tabu time because it determines the number of iterations for which a schedule maintains its tabu status. If the value of *TT* is too small, cycling may occur in the search process. By contrast, if it is too large, potential swaps may be prohibited leading to the exploration of lower quality schedules and producing a larger number of iterations to obtain the cheapest schedule. It is, however, difficult to give a general rule for determining *TT*'s best value. In this research, the best value for *TT* seems to increase as a function of the size of a network. Experiments have been carried out to determine how this value is related to the size of the network. These experimental observations show that the cycling phenomenon does not always appear with a small *TT* value. This phenomenon is in agreement with Taillard's work [38]. For the Seychelles network, smaller values, such as 3, were sufficient and cycling was not observed. The intuitive justification for this result is the fact that smaller *TT* values enable more careful examination of the cost function value space (provided that no cycling occurs). The quality of the V_{BFS} does not seem to be affected at all by fixing both the *CL* and *K* values and increasing value of the *TT* parameter from 3 to 12.

4.4.2.3. Stopping Criterion H

For investigation of the behavior of the GPS–TS technique as a function of time (iterations counts), the values of both *TT* and *CL* are fixed while increasing the value of the *H*. This parameter represents the condition whereby the technique terminates when the cost function's best value has not changed for the last tenth of the consecutive identical schedules. The increased value of *H* does not improve the value of *RRM*. The justification of this is likely to be due to the robustness of this technique and the linear type of the Seychelles network. Linear-type networks have less choice of reasonable schedules.

4.4.2.4. Extended Tabu Parameters

As discussed in the previous sections, the effect of tabu parameters was evaluated individually until no improvement could be seen. In some cases, it may be desirable to diversify the search by changing the tabu parameters simultaneously. The broader objective is to continue to stimulate the discovery of new high quality schedules by combining all the effects of the tabu parameters together. This may be achieved by increasing the number of iterations or by changing the *TT* and *CL* values during processing. Applying these criteria to the Seychelles network, no gain in performance was achieved.

For a large network, such as the GPS Seychelles network, the neighborhood has many sessions and these sessions may be costly to examine. The aggressive nature of TS makes it highly important to isolate a candidate subset of the neighborhood, and to examine this subset instead of the entire neighborhood. In conclusion, the experimental observation shows that the GPS–TS technique is quite robust. The quality of the V_{BFS} does not seem to be affected by different initial choices of parameters.

5. CONCLUSION

The best designs for the GPS–SA and GPS–TS heuristic techniques have been identified and good solutions for the Seychelles network have been found. The performance of the two techniques are evaluated with respect to computation efforts and the quality of the best schedule found by each technique. Another evaluation criterion was adopted is to monitor the progress of the search of each technique from the initial schedule until stopping criteria are met. Also, the progress of the cost of the best schedule is investigated as a function of control parameters. In comparison with GPS–SA, GPS–TS appears to be superior both with respect to the quality and computational efforts needed to find the best possible schedules. The calculation process is much easier in the GPS–TS technique compared with the GPS–SA technique (*i.e.* there are no probabilities, no exponential functions, no random decisions, *etc.*). In the GPS–TS procedure, the use of the candidate list strategy for combinatorial search offers better ways to save computational effort without affecting solution quality. In the GPS–SA technique it is crucial to provide a good initial starting schedule and carefully chosen annealing parameters, while the GPS–TS technique is insensitive to choosing the tabu parameters and an initial schedule. Both techniques have been coded in Visual C++ and implemented on a PC (Viglen p5/133).

For future work, another development to be considered is the visualisation of the program to make it user-friendly. A visual display of the network design allows the surveyor to interact with the graphical network representation and this has many advantages. Such advantages include enhancing the surveyor's understanding, making it easier to create an initial starting and feasible schedule, enabling the surveyor to determine when it makes sense to the search and detect when improvements are unlikely. It is clear that such a visual display could be more intuitive than mathematics notation, especially for the surveyor who does not have a background in heuristic techniques.

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