Ants Can Successfully Design GPS Surveying Networks

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A common problem in making measurements on a GPS surveying network with a limited number of receivers is deciding the best order in which to visit the sites and carry out the observations. The optimum site occupation schedule would be the one which provides the best results with a minimum cost in time. How does one go about finding the best schedule? It seems that ants know how. In this month's column, Dr. Hussain Saleh of the Institut de Recherches Interdisciplinaires et de Developpments en Intelligence Artificielle (IRIDIA) of the Université Libre de Bruxelles in Brussels, Belgium explains how ants efficiently find their food using an indirect communication procedure and how their approach can be mimicked in designing GPS surveying networks. Dr. Saleh is currently a postdoctoral researcher at IRIDIA working on the optimization of GPS technology using artificial intelligence. He obtained a B.Sc. degree in civil engineering from Tishreen University, Syria in 1987 and spent the next five years as an executive site engineer on projects of the Syrian Ministry of Housing and Utilities. He received an M.Sc. degree in 1996 from University College London and a Ph.D. from the University of East London in 1999. In October 2000, Dr. Saleh was awarded a European Community Marie Curie Fellowship for research within the Improving Human Research Potential Programme.

With the recent growth in the use of highly accurate geodetic surveying techniques, primarily due to the widespread adoption of GPS by the surveying community, comes the desire for a general framework for the optimal design of GPS surveying networks. GPS allows us to perform precise positioning at a fraction of the cost required by traditional methods. However, the time and cost of achieving this precision on networks can only be optimized if the logistics of the GPS fieldwork are properly investigated.

In this article, we will examine how the concept of the *ant colony system* (ACS) metaheuristic algorithm has been successfully applied to optimizing the logistics of the GPS surveying network problem. A metaheuristic technique is an iterative, self-learning procedure for quickly and efficiently identifying a high quality solution for difficult optimization problems. The ACS algorithm is inspired by the foraging behavior of ants, who can find short paths from their nest to food sources by laying down pheromone traces on the ground.

The ACS algorithm, which is a particular instance of *ant colony optimization*, has been applied successfully to a variety of complex combinatorial optimization problems (COPs). The aim of a COP is to search and determine the most suitable solution for optimizing (minimizing or maximizing) an objective function (cost, accuracy, time, distance, etc.) over a discrete set of feasible solutions. Typical examples of practical COPs are the quadratic assignment, graph coloring, job-shop scheduling, sequential ordering, and vehicle routing problems. These problems arise in business, engineering, industry and many other areas.

The designing of a GPS network as a COP consists of establishing a set of feasible schedules and then determining which one is the best schedule and costs the least. Exact methods can solve the problem for small networks (5-15 stations). However, as the size of the network increases, the number of calculations required to produce an exact solution increases exponentially and, therefore, an exact method soon becomes impractical. Metaheuristic techniques, on the other hand, can provide an optimal schedule, or close to it, for networks of up to thousands of stations with a reasonable degree of computational effort. This article proposes the ACS metaheuristic technique as a tool for finding the best schedule in occupying a GPS surveying network.

The GPS Network Problem

Within the framework of the COPs, the GPS surveying network problem can be briefly defined as follows. A number of receivers (identified as X, Y, Z, etc.) are placed at stations (a, b, c, d, etc.) to determine baselines between stations in a sequence of observing sessions (ab, ac, dc, etc.) as shown in **Figure 1**. This process of session observation continues until the whole network is completely observed. The challenge is to search for the best order in which these sessions can be organized to give the best schedule at minimum cost in elapsed time between station occupations. Mathematically, this is expressed as

$$\textit{Minimize}: C(V) = \sum_{p \in R} C(S_p)$$

where

- S_p : the route of receiver p in a schedule;
- $\Sigma C(S_p)$: the total cost of carrying out the survey of the whole network using all the receivers;
- $C(V)$: the total cost in time of a feasible schedule V.
- N : the set of stations N = {1,…,n};
- R : the set of receivers R = {1,...,r};
- U : the set of sessions U = {1,...,u}.

Metaheuristic Techniques

Interest in metaheuristic techniques developed dramatically in the early 1980s and they have had widespread success in solving a variety of difficult practical COPs. Metaheuristics, which is based on ideas from artificial intelligence, apply to a wide range of important problems in various disciplines, such as statistics, engineering, mathematical programming, and operational research. (The word heuristic comes from the Greek *heuriskein*, "to find or discover." It is used to describe problem solving techniques that use self-learning.)

The fundamental concepts of any metaheuristic technique are: representation and construction of an initial solution; generation of neighbouring solutions; acceptance strategy (that is, the criteria

and methods used to select acceptable solutions); and stopping criteria (that is, the criteria used to stop the search, such as when there is no improvement in the solution for several successive iterations, or stopping the procedure after a predefined number of iterations). Within the frame of the combinatorial optimization of GPS network scheduling, metaheuristic techniques can be classified as schedule improvement techniques and schedule construction techniques. Schedule improvement metaheuristic techniques start from a given schedule and attempt to minimize its cost through an iterative procedure until it meets the stopping criteria for the best possible solution. Schedule construction metaheuristic techniques build a feasible schedule from scratch and then attempt to minimize its cost by perturbing the schedule until it meets the stopping criteria for the best possible solution. For the work described in this article, we adopted the ACS technique, which is the most recent nature-inspired metaheuristic technique, to construct an initial schedule from scratch and then improve it.

Ants and Algorithms

The behavior of foraging ants provides the basis for an algorithm that we describe in this column, which we use to optimize GPS network design. Using a probabilistic trial-and-error method, an ant colony develops a network of optimal routes (shortest, fastest) to food sources. Foraging ants mark their path by laying down chemical cues called pheromones, varying the amount of chemical deposited depending on the quantity of food and its distance from the nest. Ants that discover the shortest route are able to move back and forth between their nest and the food source, depositing higher levels of pheromone as they do so. Another ant encountering a pheromone trail will tend to follow it, thereby reinforcing it with additional pheromone.

This behavior is a form of autocatalytic behavior: the more the ants follow a trail, the more attractive that trail becomes. This behavior of real ants has inspired the technique of ant colony optimization (ACO) developed by IRIDIA's Marco Dorigo, which has been successfully applied to many COPs.

The basic idea underlying the ACS algorithm is the use of a positive feedback mechanism that searches for the best possible solution for a hard optimization problem. Results obtained with ant-based algorithms are often as good as those obtained with other general-purpose metaheuristics.

The ACS, which is a particular instance of the ACO, uses a colony of virtual ants that behave as co-operative agents in a mathematical space in which they are allowed to search and reinforce pathways (solutions) in order to find the optimal ones. These pathways might contain very complex information. Our artificial ants have some memory and are not completely blind. Also, we consider time to be discrete rather than continuous. In the framework of the ACS algorithm, when each ant completes a tour, the pheromone along the ant's path is reinforced according to the quality of the solution the ant has found.

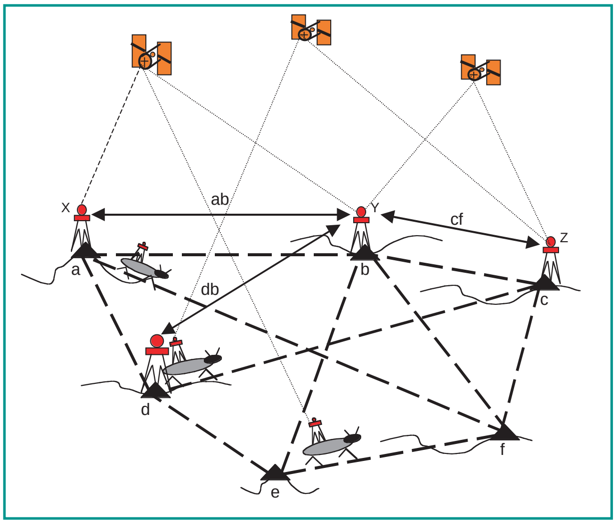

FIGURE 1 Observation of sessions using GPS receivers.

ACS Algorithm

To illustrate the procedure of the ACS algorithm, we use a small GPS network, as **Figure 2** shows, with the following symbols:

- G = (U, S) is a network (or "graph" in the context of graph theory)
- U = {1,..,u} is a set of nodes (GPS sessions)
- S = {1,..,s} is a set of paths connecting nodes (geometric distance, cost, time, or some other measures);
- $d_{ij} = [(x_i-x_j)^2+(y_i-y_j)^2]^{1/2}$ is the Euclidean (straight-line) distance between nodes i and j (that is, path ij). In the GPS network problem, this distance is represented by the time required to move from one session to another and the information is kept in the form of a cost matrix $\boldsymbol{C}$.
- M = (1,..,m) is a number of ants; the search process starts with one ant per node.

Whereas real ants deposit pheromone on the paths they visit, the algorithm's virtual ants, represented by search indices, change numeric information ("virtual pheromone") stored in variables representing the sessions they observe. This information can be changed by any ant accessing or observing the session. The shortest route (cheapest GPS schedule) is obtained by following a four-step procedure.

Starting the Algorithm. At each time step, ants construct their schedules by iteratively adding new sessions to the current partial schedule. The selection process of the next session to be observed is based on the heuristic information, $\eta(i,j)$,and pheromone level, $\tau(i,j)$, on the path connecting these two sessions, as Figure 2 shows. Heuristic information, which is based on the cost matrix, $\boldsymbol{C}$, represents the nearest sessions around the current session, while the pheromone level "memory" of each path represents the usability of this path in the past to find good schedules.

Starting from the initial session i, an explorer ant m chooses probabilistically to observe session j next, using the following transition rules:

$$i = \begin{cases} \operatorname{argmax}_{k \in S_m(i)} \left[\tau(i,k)\right]^{\alpha} \cdot \left[\eta(i,k)\right]^{\beta} & \text{if } q \le q_0 \\ I & \text{otherwise} \end{cases} \quad (1)$$

[[Where is "k" defined?]]

where I is a random variable selected according to the distribution given by Equation (2) which gives the probability that an ant in session i chooses to move to session j as follows:

$$P_m(i,j) = \begin{cases} \dfrac{\left[\tau(i,j)\right]^{\alpha} \cdot \left[\eta(i,j)\right]^{\beta}}{\sum_{k \in S_m(i)} \left[\tau(i,j)\right]^{\alpha} \cdot \left[\eta(i,j)\right]^{\beta}} & \text{if } j \in S_m(i) \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

where

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⊕ $\tau(i,j)$ is the intensity measure of the amount of pheromone deposited on the path (i,j) by each ant. The higher the level of pheromone on a given path, the more likely that an explorer ant will take that path. The trail changes during each step of the run of the ACS program.

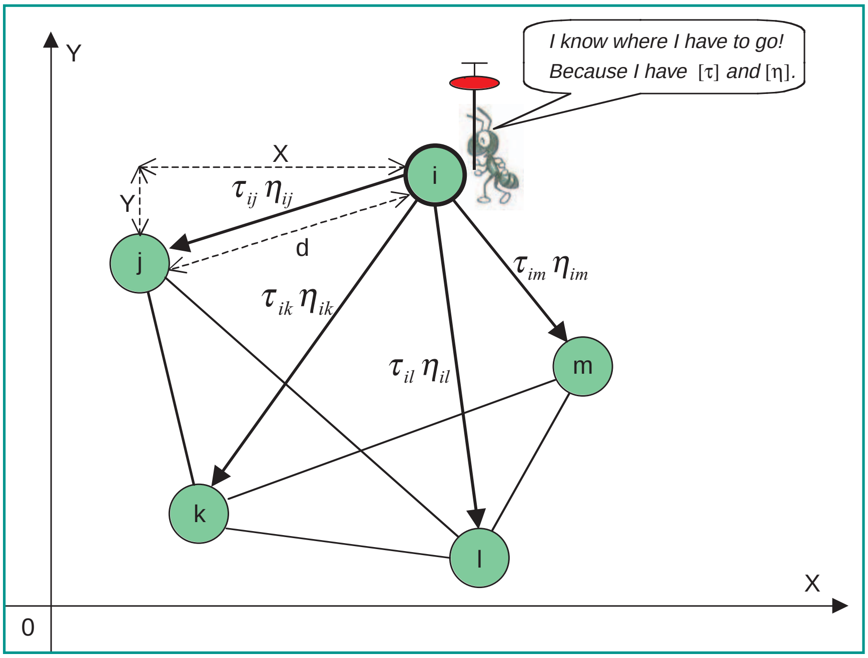

FIGURE 2 Selection process of the GPS sessions using the ACS concept.

⊕ α is is a weighting parameter to control the intensity.

⊕ Visibility $\eta(i,j) = 1/C_{ij}$, is a heuristic function given to an ant to make some local decisions and possibly opt for the nearest session at a given step in the process (heuristic desirability). The visibility remains constant during the run of the ACS program.

⊕ β is a weighting parameter to control the degree of visibility.

⊕ q is a uniformly distributed random number $q \in [0, 1]$ and its purpose is to determine the relative importance of exploitation versus exploration.

⊕ q_0 is a threshold parameter and the smaller q_0 the higher the probability to make a random choice.

⊕ $S_m(i)$ is the set of sessions that remain to be observed by ant m positioned at session i.

As the preceding probability equations show, the purpose of constants α and β is to control the sensitivity of the search to pheromone concentration and link cost, respectively. This probability is based on the travel time to the nearest sessions that the ant has not yet observed, as well as on the amount of pheromone present at that moment on the different allowed paths. Therefore, the transition probability is a trade-off between the value of the visibility function (travel time) and trail intensity (amount of pheromone) at time t. Visibility means that the nearer sessions should be selected with high probability (greedy constructive heuristic), while the trail intensity means that path (i,j) is highly desirable if it has seen a lot of traffic (positive feedback).

In each step of building a schedule, an ant located at session i samples the parameter q to decided whether to move to session j. Using Equation 1, an ant selects the best path to observe the next session when $q < q_0$ (exploitation). Otherwise, the ACS algorithm will probabilistically choose the next session to be observed using Equation 2 with a bias toward the best possible path (biased exploration). The above equations show that the quality of the path (i,j) is proportional to its shortness and to the highest amount of pheromone deposited on it (that is, the selection probability is proportional to path quality).

Local Search Method. One must design a local search procedure that suits the GPS network requirements and add it to the main ACS algorithm. The main

purpose of implementation of the local search method (LSM) is to speed up and improve the solutions constructed by the metaheuristics techniques.

The LSM (a move-by-move method) perturbs a given solution to generate a different neighbouring solution using a move generation mechanism. This mechanism, which is based on the exchange of the solution's components, is a transition from one solution to another in one step.

Within the context of GPS surveying, the LSM attempts to improve a given schedule by a series of local improvements (swapping sessions). We have developed and implemented the most suitable local search structure that satisfies the GPS requirements. In this sequential structure, which is based on the sessions-interchange, the potential pair-swaps are examined in the order (1,2), (1,3),…, (1,n), (2,3), (2,4), (u-1, u), etc. The change in cost is computed and the swap is accepted or rejected according to the acceptance strategy of the implemented optimization technique. The basic steps for the LSM are as follows:

- Select a given schedule V ϵ I(V) and compute its cost value C(V).
- Generate a schedule V´ ϵ I(V) and compute its cost value C(V´).
- If C(V´) < C(V) then replace V with V´ as a current schedule.
- Otherwise, retain V and generate other moves until C(V´) < C(V) for all V´ϵ I(V).
- Terminate the search and return V as the local optimal schedule.

The Local Updating Rule. While ants build their schedule, at the same time they locally update the pheromone level of the visited paths by applying the local updating rule (LUR) as follows:

$$\tau(i,j) \leftarrow (1-\rho)\cdot\tau(i,j)+\rho.\tau_0 \qquad (3)$$

where

- ρ is a persistence of the trail and the term (1-ρ) can be interpreted as trail evaporation;
- τ_0 is the initial pheromone level which is assumed to be a small positive constant distributed equally on all the paths of the network.

The aim of the LUR is to make better use of the pheromone information by dynamically changing the desirability of paths. Using the LUR, ants will search near the best previous schedule. As shown in this equation, the pheromone level on the paths of the constructed schedule is highly related to the value of evaporation parameter ρ. The pheromone level will be reduced when ρ has a big value and this will reduce the chance that the other ants will select the same schedule and consequently the search will be more diversified. Therefore, care must be taken when choosing the value of ρ to balance the search process.

The Global Updating Rule. When all ants have completed their schedules, the pheromone level is updated by applying the global updating rule (GUR) only on the paths that belong to the best schedule since the beginning of the trail, as follows:

$$\tau(i,j) \leftarrow (1-\varphi)\cdot\tau(i,j)+\varphi\cdot\Delta\tau(i,j) \qquad (4)$$

where

$$\Delta\tau(i,j) = \begin{cases} (C_{GBS})^{-1} & if(i,j) \in \\ Global - Best - Schedule & \\ 0 & otherwise \end{cases} \quad (5)$$

and where

- C_{GBS} is the cost of the best found schedule from the beginning;
- $\Delta\tau(i,j)$ is the pheromone level on path (i,j); and
- φ is an evaporation parameter.

The GUR rule is intended to provide a greater amount of pheromone on the paths of the best schedule, using ϕ to intensify the search around this schedule. In other words, only the best ant that took the shortest route between sessions is allowed to deposit pheromone.

Implementation of the Algorithm

To improve the search process, we must carefully choose the components when applying the ACS algorithm to the GPS network problem. These components, which must be defined according to the size and type of the network to which they are to be applied, are the structural elements and control parameters.

The *structural elements* determine the procedure in which the GPS network is modelled in order to fit the ACS framework. They consist of: the cost matrix (travel time between sessions), the cost function (total time ΣC_{ij} to complete the observation of all the required sessions in the network); the number of ants and their initial starting locations (the number of ants can be set equal to the number of the sessions to be observed and ants can be located randomly or selectively for the initial sessions); and the taboo (spelled tabu in the mathematical literature) list (associated with each ant in order to prevent it from visiting a session more than once).

The *control parameters* govern the workings of the ACS technique itself and are mainly concerned with pheromone information. They consist of: the pheromone intensity control parameter α, the visibility control parameter β, the evaporation control parameters ρ and φ, and the stopping criterion (which, in the current implementation of the technique, terminates the process after a pre-defined number of iterations). Recall that parameter φ is used in the global updating rule to direct the search by adding the pheromone on the paths of the best-obtained schedule. Parameter ρ is used in the local update rule to diversify the search by removing some pheromone from the paths of the current schedule.

The basic ACS technique produces only the best-found solution from the beginning which satisfies the stopping criteria. The modified GPS-ACS technique adopts a candidate list which contains extra alternatives for the obtained good schedules as well as the best one. The number of these alternatives (that is, the length of the candidate list) is dependent on the user.

The main objective of this list is to give alternatives for the surveyor to select the best schedule which satisfies all the GPS field-work requirements. The candidate list is a data structure which contains the best possible schedules found from the beginning of the search process. At the end of each iteration, the algorithm compares the contents of the new candidate list for the current iteration with

Solving the GPS Network Problem Using the Ant Colony System Algorithmic Procedure

[I] INITIALISATION

(A) FORMULATING the original cost matrix:
{Original cost matrix represents the cost of moving the receiver from one station to another}
insert the total number of stations, n;
insert the estimated cost for each receiver's move.
(B) CREATING the actual cost matrix:
{Actual cost matrix represents the cost of moving the receiver from one session to another}
insert the number of receivers, r;
define the sessions to be observed, u.
(C) DETERMINING the structural elements:
set the number of ants M (M=U);
set the tabu list for each ant;
set the candidate list for the best found schedules;
set the initial pheromone τ_0.
(D) INITIALIZING the control parameters:
set the trail control parameter α;
set the visibility control parameter β;
set the evaporation control parameters ϕ and ρ;
set the iteration counter, K=0.

[II] SELECTION AND ACCEPTANCE STRATEGY

(E) SELECTING the best admissible move of cost $C(V_{best})$:
build up the cheapest schedule by applying (u-1) moves;
select the next potential move using the probability equations (Equations 1 and 2);
add the observed sessions by each ant to its tabu list as it proceeds, then empty the tabu list when the schedule is completed;
apply the local search method.
(F) UPDATING the pheromone level:
apply the local update rule for each path as each ant proceeds till the schedule is completed using (Equation 3)
apply the global update rule on the paths of the best found schedule from the beginning using (Equations 4);
update the counter K=K+1, and the process continues.

[III] THE STOPPING RULES

(G) TERMINATING the search:
stop if the stopping criterion is satisfied;
given number of iterations, OR
maximum number of iterations allowed without improving the best obtained schedule.
OTHERWISE
Go to (E).
(H) DECLARING the output:
declare the best obtained schedule;
declare the computation time;
END.

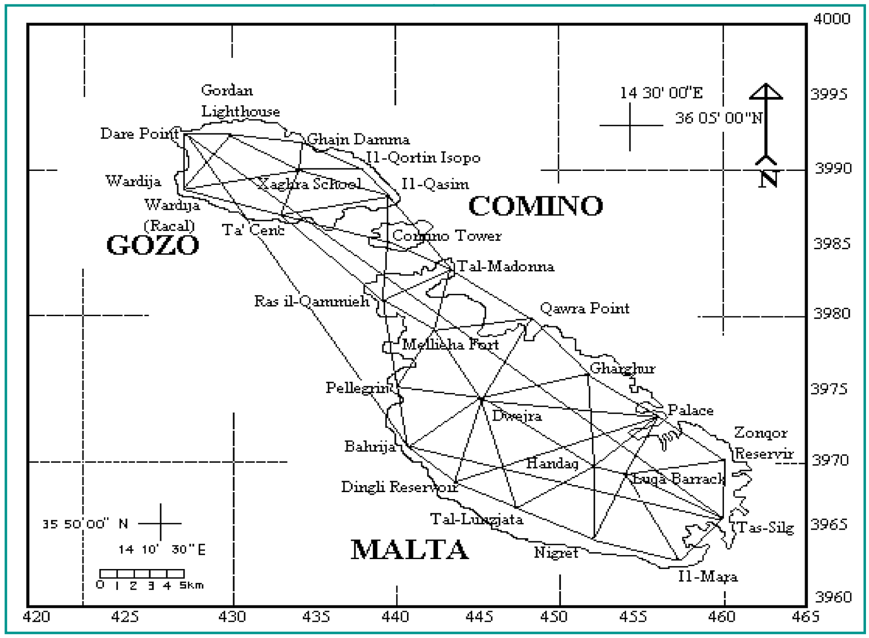

FIGURE 3 Malta GPS surveying network.

the contents of the old candidate list for the previous iteration and then selects the best schedules among all the others for further comparison. This process of comparisons to select the best candidate list continues through the search process until the algorithm meets the stopping criteria. At the end of the search process, the algorithm keeps in the final candidate list the best schedules found from the beginning.

In this algorithm, each ant will construct one possible design of the whole GPS network. During an initialization phase, ants are randomly positioned on different sessions with empty tabu lists and the paths connecting the sessions to observe have initial values for trail intensity. The schedule construction stage starts when every ant chooses to move from session *i* to session *j* with a probability which is a function of the intensity and visibility measures.

Each time an ant makes a move, it leaves a pheromone trail on the connecting path which then will be collected and used to compute the new values for the transition probabilities according to Equations (1) and (2). After (u−1) moves, ants complete their schedules in which the tabu list of each ant will be full. The cheapest schedule found is computed and memorized; then tabu lists are emptied for the next iteration. The algorithm iterates the optimization process for the proposed ACS technique until the stopping criteria to find the best possible schedule are met. The **sidebar** "Solving the GPS Network Problem Using the Ant Colony System Algorithmic Procedure" shows the structure of the ACS algorithm for optimizing the GPS network problem.

Computational Results

In metaheuristics it is preferable to evaluate the performance of a proposed technique by comparison with an existing optimal solution. This section assesses the effectiveness of the ACS algorithm and compares its performance with respect to solution quality and computational effort with that of an alternative technique. A common measure to define the quality of the ACS solution is the relative percentage deviation (RPD) from the optimal solution, which can be computed as follows:

RPD = [ACS Solution - Optimal Solution/(ACS Solution)] × 100.

Peter Dare, now teaching at the University of New Brunswick, obtained some known optimal schedules for relatively small GPS surveying networks. The first one was a hypothetical GPS network and consisted of six sessions. The cost of the metaheuristic schedule obtained using the proposed ACS technique was the same as the known optimal schedule. The process of the ACS was terminated when the stopping criterion was met after a defined number of iterations, $k = 8$.

The selected control parameters for this network were $\alpha = 1$, $\beta = 1$ and $\rho = \varphi = 0.3$. The second network was an actual GPS network observed in New

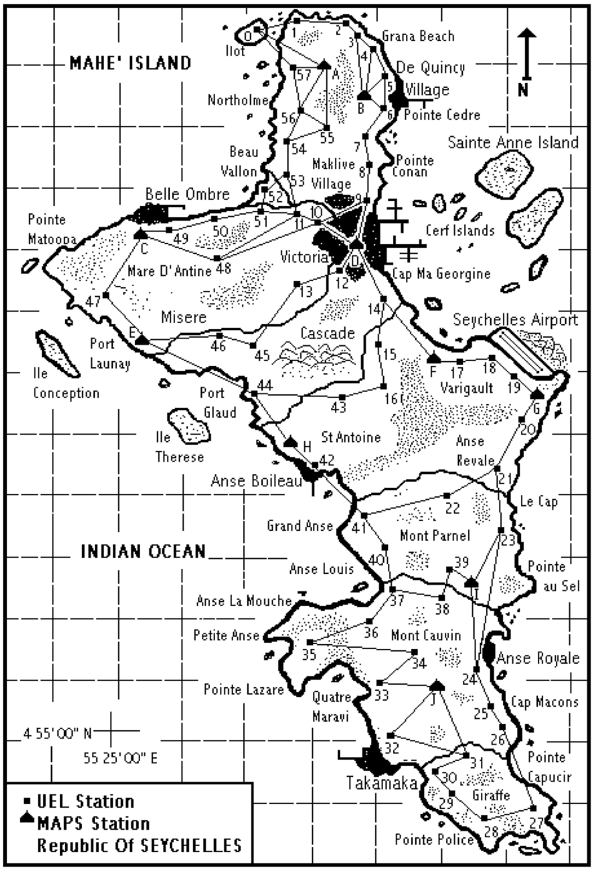

FIGURE 4 The Seychelles GPS surveying network.

Brunswick, Canada and consisted of ten sessions. By applying the ACS technique, the same cost as the optimal schedule was obtained and the technique stopped when there were no further im-provements after $k = 10$ iterations. The selected control parameters for this network were $\alpha = 1$, $\beta = 1$ and $\varphi = 0.6$, and $\rho = 0.3$.

To generalize the developed ACS technique and work with larger networks, we used two different types and sizes of GPS surveying networks. The first one is a triangulation-type network on the island of Malta consisting of 38 sessions and 25 stations as shown in **Figure 3**. The second one is a linear-type network in the Seychelles, consisting of 71 sessions and 57 stations, as shown in **Figure 4**.

Several benchmarks or reference schedules were available for these networks, which allowed for comparisons with respect to the effectiveness and computational efficiency of the proposed ACS technique. For the Malta network, the first benchmark represented the actual operating schedule with a cost of 1,405 minutes. The surveyors carrying out the work used their intuition and experience to manually generate this schedule. The other two benchmarks were the metaheuristic schedules obtained by applying simulated annealing and tabu search techniques and had a cost of 1,355 minutes and 1,075 minutes respectively. By implementing the ACS technique using the same data set of 38 sessions, the overall cost to observe the Malta GPS network was reduced to 895 minutes after 80 iterations. The selected control parameters for this network were $\alpha = 1$, $\beta = 2$ and $\varphi = 0.7$, and $\rho = 0.4$.

For the Seychelles GPS network, the first benchmark, which was manually generated, represented the actual operating schedule with a cost of 994 minutes. The other two benchmarks were the metaheuristic schedules obtained by applying simulated annealing and tabu search techniques and had a cost of 976 minutes and 933 minutes respectively. By implementing the ACS technique using the same data set, the overall cost to observe the Seychelles GPS network was reduced to 853 minutes after 100 iterations. The selected control parameters for this network were $\alpha = 1$, $\beta = 2$ and $\varphi = 0.6$, and $\rho = 0.3$.

Comparative Analysis

The main goal of the developed metaheuristic techniques is to minimize the overall observation times of the GPS networks. The most useful measure for the evaluation the performance of these techniques is the *relative reduction of the cost* (RRC) provided by these techniques with respect to the actual operational schedule, that is,

$$RRC=[(C_{OS}-C_{ACS})/C_{OS}] \times 100$$

Figure 5 and **Figure 6** illustrate the RRC achieved by the GPS-ACS technique for the Malta and the Seychelles networks compared to the simulated annealing and tabu search techniques. The achieved results, listed in **Table 1**, indicate that the developed GPS-ACS metaheuristic technique consistently produced better schedules.

We experimented with different parameter settings to investigate the impact of the control parameters on solution quality. When we tested the value of one parameter, we set the others at their default value, namely $\varphi = 0.5$, $\rho = 0.3$, $\beta = 2$ and $\alpha = 1$. The values of parameter β leading to better results vary widely between 1 and 3. Parameter β is used to adjust the relative importance of pheromone traces when evaluating the cost of a path.

With regards to parameter φ, good results were obtained with values smaller than 0.6. On the other hand, larger values of ϕ tend to bias the search process toward elite schedules and lead to parameter convergence.

For parameter ρ, better results were found when $\rho = 0.3$ which indicate that a significant level of diversification is desirable.

To test our ideas and to try them out on hypothetical and real networks, we coded all the developed algorithms in the Visual C++ programming language.

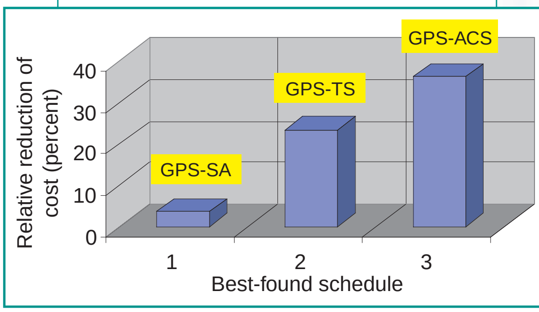

FIGURE 5 The RRC for the best schedule for observing the Malta GPS network obtained by the simulated annealing (SA), tabu search (TS) and ant colony system (ACS) GPS metaheuristic techniques.

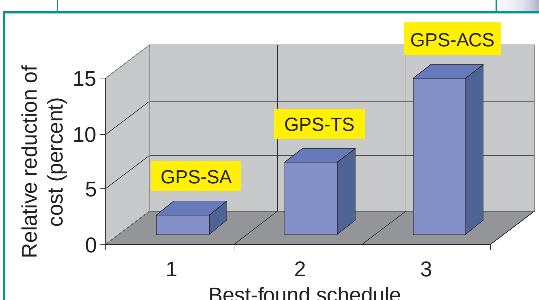

FIGURE 6 The RRC for the best schedule for observing the Seychelles GPS network obtained by the simulated annealing (SA), tabu search (TS) and ant colony system (ACS) GPS metaheuristic techniques.

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TABLE 1 Comparison of GPS Metaheuristic Techniques on Malta and Seychelles Networks

Network Information			GPS Metaheuristic Techniques								
			GPS-SA			GPS-TS			GPS-ACS		
Network	U	C_{OS}	C_{SA}	K	ET	C_{TS}	K	ET	C_{ACS}	K	ET
Malta	38	1,405	1,355	14,880	425	1,075	28	6	895	80	44
Seychelles	71	994	976	115,920	1,700	933	20	40	853	100	153

where
- N is the number of sessions.
- C_{OS} is the cost of the operational solution created by an experienced surveyor.
- C_{SA} is the cost of the metaheuristic solution created by simulated annealing technique.
- C_{TS} is the cost of the metaheuristic solution obtained by tabu search technique.
- C_{ACS} is the cost of the metaheuristic solution obtained by ant colony system technique.
- K is the number of iterations.
- ET is the execution time in seconds.

All costs are in minutes.

Conclusion

We have successfully applied the ACS metaheuristic technique to the GPS surveying network problem and obtained good results for networks consisting of up to 57 stations. Future research should include the analysis of parameter settings for even larger GPS networks. Another important direction for research in this area is to add different local search strategies to the main program to explore the schedule space more effectively and provide good results.

Another application in which the metaheuristic techniques can be implemented is the optimization of ambiguity resolution in GPS data. GPS carrier-phase measurements contain an unknown number of integer wavelengths which biases all measurements in an unbroken sequence of satellite observations. This unknown number of integer wavelengths, called integer ambiguity, must be determined correctly to quickly provide high precision GPS positioning. However, resolving GPS ambiguity problem is a complex task when high performance and computational efficiency are required. The research on metaheuristic techniques is a promising direction for producing an effective time-efficient solution to this problem.

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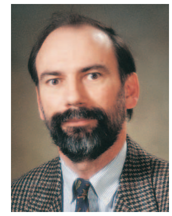

"Innovation" is a regular column featuring discussions about recent advances in GPS technology and its applications as well as the fundamentals of GPS positioning. The column is coordinated by Richard Langley of the Department of Geodesy and Geomatics Engineering at the University of New Brunswick, who appreciates receiving your comments as well as topic suggestions for future columns. To contact him, see the "Columnists" section on page 2 of this issue.

Further Reading

For general introductions to metaheuristic techniques, see

- "Meta-heuristics: An overview" in *Meta-heuristics: Theory and Applications* edited by I. H. Osman and J. P. Kelly, published by Kluwer Academic Publishers, Dordrecht, The Netherlands, 1992.
- *Reinforcement Learning, an Introduction*, edited by R.S. Sutton and A.G. Barto, published by MIT Press, Cambridge, Massachusetts, 1998.
- Metaheuristics Network <http://www.metaheuristics.org/>

For further details on the ant colony system, see

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